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MODELING OF THE DEVICE AND METHODS FOR MEASURING THE TURBULENT PRANDTL NUMBER IN A FLUID FLOW©

The table of symbols

a - thermal diffusivity, m^2/s;	*εa* - thermal turbulent diffusivity;
L, Zmax - length of heat-exchange tube division;	*εσ* - kinematic turbulent viscosity;
ℓ - grid pitch along axis Z;	*εσ/ν* - turbulent transfer momentum;
Pr - molecular Prandtl number;	*η* - dimentionless distance from a wall;
Prtb - turbulent Prandtl number;	*Θ* - dimentionless temperature;
Rmax - internal tube radius, m;	*λ* - thermal conductance, W/(m˚C);
r - radial coordinate, m;	*ν* - kinematic fluid viscosity, m^2/s;
Tin - initial fluid temperature, ˚C;	*ξ* - friction drag;
Tc - temperature in the upper heat exchanger, ˚C;	*ρ**i*** - grid pitch along axis ***r***;
Tb - bulk fluid temperature at a tube end, ˚C;	υ* - dynamic velocity, m/s;
U - averaged fluid temperature, ˚C;	*φ* - dimentionless velocity;
z - axial coordinate, m;	¯ - average fluid velocity along tube cut, m/s;
	*ω**z*** - averaged fluid velocity in the line of axis z, m/s.

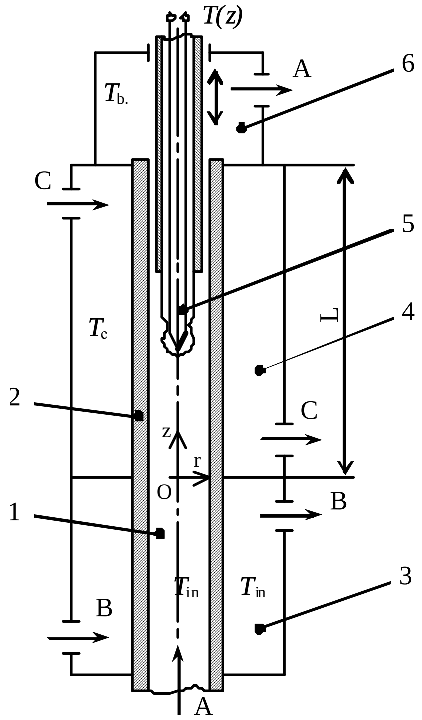
Introduction

Turbulence - is a very complex physical phenomenon, which is insufficiently examined, though much theoretical work and experimentation has been carrying out [7]. The notion of turbulent fluid flow is generally defined as a moving, when in a flow there are pressure pulsations and random velocity, which mix it. Besides, unlike laminar flow regime, an irregular mode of flow quantity changing is emphasized. A chaotic manner of changing characteristics makes it both impossible and unpractical to obtain the mechanisms, which describe the change of instant quantity values. From practical point of view, averaged quantity values of turbulent flow are the most interesting.

One of these averaged values, characterizing turbulent flow, is the so-called turbulent Prandtl number ***Prtb***, equal to the ratio of kinematic turbulent transfer momentum *εσ* (kinematic turbulent viscosity) to the turbulent heat transfer εq (thermal turbulent diffusivity): $\mathrm{Pr}_{тб} = \;/\;_{q}$. In contrast to the molecular Prandtl number, ***Prtb*** is not a physical characterstic of medium. ***Prtb*** number changes along the flow cut, moreover it depends upon Rainold's number ***Re*** and molecular Prandtl number ***Pr***. A lot of experimentation deals with the research of turbular Prandtl number, according to which ***Prtb*** differs from a unit not much. That is why when making theoretical calculations, Rainold's hypothesis is commonly used as the first approximation, under which the turbulent heat transfer and turbulent transfer momentum are equal. Though, it is necessary to point out that available at present experimental data on turbular Prandtl number ***Prtb*** (***r***, ***Re***, ***Pr***) and the character of its changing are very contradictory [Ibidem].

In this paper the two methods and the device of turbulent Prandtl number detection in fluid flow are put forward. These methods can be successfully used to investigate the turbulent Prandtl number dependence of Rainold's number and of molecular Prandtl number, what in turn will assume to define the scope of Rainold's hypothesis and to estimate the accuracy, occurring when ***Prtb*** is rigidly taken as ***Prtb*** = 1.

1. The structure of measuring the equipment



Picture 1. The physical model of the device

An experimental device for turbulent Prandtl number detection (Picture 1) includes a copper tube 2, where fluid 1 with certain initial temperature ***Tin*** flows; a lower heat-exchanger 3 (inside which heat-carrying fluid with temperature ***Tin*** flows); an upper heat-exchanger 4, inside which heat-carrying fluid flows with temperature ***Tc***; an arrangement 5 to measure fluid temperature at different tube cuts 2, which can either be lowered to depth, corresponding to the beginning of heat-exchanging site and will measure the initial fluid temperature ***Tin*** as well, or can be lifted into a tank 6 and will even measure fluid bulk temperature ***Tb*** at a tube end.

Fluid flow 1 is supposed to be turbulent and its average velocity ¯ is known and doesn't change during the experiment. Fluid heat-transfer properties, such as thermal conductance *λ*, thermal diffusivity ***a*** and kinematic viscosity ***v***, must also be known. A tube site 2, coming through the lower heat-exchanger, is used to regulate flow profile (its averaged characteristics). When the flow is turbulent, the initial site length amounts from 50 to 100 tube diameters according to G. Kirsten's measurements [5], while Nikuradze's measurements [6] amount from 25 to 40 diameters. Thus, when the internal tube diameter is 10 mm, the initial site length must not be less than 500 mm.

Using this arrangement, it is possible to define turbulent Prandtl number at least in two ways:

- First, as the numerical solution of the direct boundary (-volume) problem is matched, describing heat transfer in the fluid flow, when bulk fluid temperature at a tube end corresponds to a certain temperature, obtained from the experiment.
- Second, as the solution of Sturm-Liouville boundary (-volume) problem is matched (which arises when the solution of the premise direct boundary (-volume) problem is matched by variables separation method), when its first eigenvalue of this boundary (-volume) problem is equal to the corresponding value, measured by the temperatures, obtained at several tube cuts during the experiment.

Each of these methods has advantages and disadvantages. While applying the first method it's necessary to measure the bulk fluid temperature ***Tb*** at a tube end 2. The temperature can be measured with ultimate accuracy. While applying the second method it's necessary to measure the temperature at two different tube sections (when $z=z_1$ and $z=z_2$), furthermore both times, the point of temperature measuring must lie at regular intervals from the tube longitudinal axis. Though the second method is more complex, the outcome of the experiment is processed much faster, because in this case one-dimensional problem is solved unlike the first method, when it's required to find the solution to one-dimensional problem.

2. The mathematical model of the device

The mathematical model, describing temperature distribution in a turbulent fluid flow by means of averaged fluid speed and temperature, includes differential equation of energy and boundary conditions. The differential equation subject to the assumptions, accepted in [7], is written in the following form:

$$\frac{\ _z(r)}{\ }\frac{\partial U}{\partial z}(r,z)=\frac{1}{r}\cdot\frac{\partial}{\partial r}\left[r\left(\frac{1}{\mathrm{Pr}}+\frac{1}{\mathrm{Pr}_{\text{тб}}}\cdot\frac{\ }{\ }(r)\right)\frac{\partial U}{\partial r}(r,z)\right] \qquad (1)$$

$0\le r\le Rmax$

$0\le z\le Zmax$

Where ***U*** - is the averaged fluid temperature; ***r, z*** - are the radial and axial coordinates; ***Rmax*** - is the internal tube radius; ***Zmax*** - is the length of a heat-exchange tube division (in the line of axis z); ***ν*** - is the kinematic fluid viscosity; ***Pr*** - is the molecular Prandtl number for the given fluid; *ωz* (*r*) - is the averaged fluid velocity in the line of axis *z* (the averaged fluid velocity in the line of axis ***r*** is taken as zero); $\frac{\ }{\ }(r)$ - the turbulent transfer momentum.

The differential equation (1) is complemented with the following boundary conditions:

$$U(r,0)=T_{\text{н}};\ \frac{\partial U}{\partial r}(0,z)=0;\ U(R_{\max},z)=T_{\text{с}} \qquad (2)$$

While applying the method of solution by means of the direct boundary (-volume) problem instead of boundary conditions 1 at inner surface of a tube wall boundary conditions 2 can be specified:

$\frac{\partial U}{\partial r}(R_{\max},z)=q_{\text{с}}$, where $q_{\text{с}}$ - is a heat flow along a tube wall, or $\frac{\partial U}{\partial r}(R_{\max},z)=\ _{\text{с}}\left[\Psi-U(R_{\max},z)\right]$, where $_{\text{с}}$ - is a heat-transfer factor at a wall, Ψ - is the temperature of a heat-transfer agent in the upper heat exchanger.

In recent years much experimentation and research of velocity fields in turbulent fluid and gas flows have been made. On the ground of this research a number of empirical and semi-empirical formulae for velocity profile and turbulent transfer momentum were advanced. While describing these quantities the so-called universal coordinates are often entered: dimensionless velocity $\ =\ _z/\ _*$ and dimensionless distance from a wall $\ =\frac{\ _*(R_{\max}-r)}{\ }$, where $_*$ - is a dynamic velocity; $_*=\bar{\ }\sqrt{\frac{\Xi}{8}}$; $\bar{\ }$ - is an average fluid velocity along a tube cut; Ξ - is a friction drag, which is defined by Rainold's number for smooth tubes with a stabilized flow $\mathrm{Re}=\frac{2\bar{\ }R_{\max}}{\ }$ and can be evaluated, for example, by the G. K. Philonenko formula $\Xi=\frac{1}{(1{,}82\cdot\lg\mathrm{Re}-1{,}64)^2}$. In addition to these quantities, dimensionless radius is entered $r_*=r/R_{\max}$.

In order to evaluate dimensionless velocity and turbulent transfer momentum both Reihardt and Disler formulae are frequently used. Reihardt formula [8] for velocity profile in a tube is of the form

$$\ =2{,}5\cdot\ln\left[(1+0{,}4\ \ )\frac{1{,}5(1+r_*)}{1+2r_*^2}\right]+7{,}8\cdot\left[1-\exp\left(-\frac{\ }{11}\right)-\frac{\ }{11}\exp(-0{,}33\ \ )\right] \qquad (3)$$

This formula describes the whole velocity profile with the single profile from the wall to the flow axis. For the turbulent transfer momentum Reihardt offered the following profile:

$$\text{when}\ \ \le 50\ \ \frac{\ }{\ }=0{,}4\cdot\left(\ \ -11th\frac{\ }{11}\right) \qquad (4)$$

$$\text{when}\ \ >50\ \ \frac{\ }{\ }=0{,}133\ \ (0{,}5+r_*^2)(1+r_*).$$

Disler [4] divides the flow into two areas and recommends the following formulae for each:

$$\text{when}\ \ <26\ \ \frac{d}{d}=\frac{1}{1+n^2\ \ \left[1-\exp(-n^2\ \ )\right]} \qquad (5)$$

$$\text{—} = n^2 \left[1 - \exp\left(-n^2 \quad\right)\right]$$, where n = 0,124;

$$> 26 \quad = \frac{1}{0,36}\ln\frac{\quad}{26} + 12,85;$$

$$\text{—} = 0,36\left(1 - \frac{\quad}{\ _0}\right) \quad -1.$$

In order to obtain explicit dependence *φ*(η) by the formula (5), it's necessary to apply one of the methods to solve an ordinary differential equation, for example, Runge-Kuttae method [2]. Its initial condition is of the form: *φ*(0)= 0 (from the condition of liquid sticking to a wall).

3. The method of finding *Prtb* from measured bulk fluid temperature at a tube end

The method of finding *Prtb* by means of the direct boundary (-volume) problem is based on fitting such value of turbulent Prandtl number, when the bulk (fluid) temperature at a tube end, equal to temperature *Tb*, and obtained during the experiment, will correspond to the temperature field $U(r,z)$, obtained just in the process of the solution of the equation (1)-(2), $U_{\text{ср.м.}}(Z_{\max}) = T_{\text{ср.м.}}$. Calculation is started when some initial estimate for *Prtb* is being chosen (it's always possible to specify *Prtb0* = 1).

In order to solve the differential equation (1) let's apply the method of finite differences. We'll inject a grid with axes *r* and *z*. The grid along the axis *r* will be non-uniform, concentrating in the line from the flow axis towards the inside wall of the tube. The grid points of the axis *r* are numbered with index $i = \overline{0,M}$ ($r_0 = 0$, $r_M = R_{\max}$). The grid pitch of the axis *r* is denoted as $\ _i$ ($\ _i = r_{i+1} - r_i$). The successive pitches are interconnected by the relation: $\ _i / \ _{i+1} = K_r$, where $K_r > 1$. The grid along the axis *z* uniform to the pitch *ℓ*; the points are numbered with the index $j = \overline{0,N}$ ($z_0 = 0$, $z_N = Z_{\max}$).

Let's pass to the equation (1) from partial differentials to finite differences, approximating this equation on the formed grid. To make it easy, let's inject a designation: $S(r) = \frac{1}{\text{Pr}} + \frac{1}{\text{Pr}_{\text{тб}}} \cdot \text{—}(r)$. Besides, we'll open the brackets in (1):

$$\frac{\ _z(r)}{\quad} \cdot \frac{\partial U}{\partial z}(r,z) = \frac{1}{r} S(r) \frac{\partial U}{\partial r}(r,z) + \frac{\partial S}{\partial r}(r) \frac{\partial U}{\partial r}(r,z) + S(r) \frac{\partial^2 U}{\partial r^2} \qquad (6)$$

Now, we'll factor out $\frac{\partial U}{\partial r}(r,z)$ in the right part and replace all the used derivatives in the equation with the corresponding finite differences:

$$\frac{\ _{zi}}{\quad} \cdot \frac{U_{ij} - U_{i\,j-1}}{\ell} = \left[-\frac{1}{2\ _{i-1}} U_{i-1\,j} + \left(\frac{1}{2\ _{i-1}} - \frac{1}{2\ _i}\right) U_{ij} + \frac{1}{2\ _i} U_{i+1\,j} \right] \times$$
$$\times \left(\frac{S_i}{r_i} + \frac{S_i - S_{i-1}}{2\ _{i-1}} + \frac{S_{i+1} - S_i}{2\ _i} \right) + S_i \left\{ \frac{2}{\ _{i-1} + \ _i} \left[\frac{1}{\ _{i-1}} U_{i-1\,j} - \left(\frac{1}{\ _{i-1}} + \frac{1}{\ _i} \right) U_{ij} + \frac{1}{\ _i} U_{i+1\,j} \right] \right\} \qquad (7)$$

The finite-difference equation (7) needs supplementing with boundary conditions, which are the analogue conditions (2):

$$U_{i0} = T_{\text{in}};\ U_{0j} = U_{1j};\ U_{Mj} = T_{\text{c}} \qquad (8)$$

The system of difference equations (7)-(8) can be easily solved by a multiple marching [6] along the axis *r* from the tube cut $z_0 = 0$ to the cut $z_N = Z_{\max}$. The result of the equation is the temperature field U_{ij} on the injected by us grid. Then we can find an estimated value of the bulk fluid temperature at a tube end. The value is calculated by the formula:

$$U_{\text{b}}(Z_{\max}) = \frac{\int_0^{R_{\max}} U(r,z) \cdot r \cdot \ _z(r)\,dr}{\int_0^{R_{\max}} r \cdot \ _z(r)\,dr} \qquad (9)$$

Passing to the notation on the grid and finding integrals by the trapezium method we obtain

$$U_{\text{b.N}} = \frac{\sum_{i=0}^{M-1} \left[\left(U_{iN} \cdot r_i \cdot \ _{zi} + U_{i+1N} \cdot r_{i+1} \cdot \ _{zi+1} \right) \cdot 0,5 \cdot \ _i \right]}{\sum_{i=0}^{M-1} \left[\left(r_i \cdot \ _{zi} + r_{i+1} \cdot \ _{zi+1} \right) \cdot 0,5 \cdot \ _i \right]} \qquad (10)$$

The value of $U_{\mathrm{b.N}}$ is compared to the value of ***Tb***, measured in the tank 6 during the experiment. If these temperatures differ more than in uncertainty of ***Tb*** measurement, the following approximation for ***Prtb*** is chosen, and then the whole calculation is repeated again. We must find the root of the equation

$$U_{\mathrm{b.N}}\left(\mathrm{Pr}_{\mathrm{tb}}\right)-T_{\mathrm{b}}=0 \qquad (11)$$

One of the quickest methods of root evaluating is the method of bipartitioning [2]. Though, for its application it's necessary to allocate the band, where the function changes the sign. For this purpose a prior scanning of the assumed field of the root evaluating with some constant pitch can be used. A similar algorithm is used as the basis of the computer program, which helps to calculate the value of ***Prtb*** from the measured bulk temperature ***Tb*** at a tube end.

4. The method of finding *Prtb* from the first eigenvalue of Sturm-Liouville boundary (-volume) problem

The direct boundary (-volume) problem (1)-(2) can be calculated by the method of variables division. Let's substitute in the equation (6), obtained from the equation (1), instead of $U(r,z)$, a dimensionless temperature $\Theta(r,z)=\frac{U(r,z)-T_{\mathrm{c}}}{T_{\mathrm{in}}-T_{\mathrm{c}}}$:

$$\frac{{}_z(r)}{}\cdot\frac{\partial\Theta}{\partial z}(r,z)=\frac{1}{r}S(r)\,\frac{\partial\Theta}{\partial r}(r,z)+\frac{\partial S}{\partial r}(r)\,\frac{\partial\Theta}{\partial r}(r,z)+S(r)\,\frac{\partial^2\Theta}{\partial r^2} \qquad (12)$$

The boundary conditions will be taken down in the form:

$$\Theta(r,0)=1\,;\ \frac{\partial\Theta}{\partial r}(0,z)=0\,;\ \Theta(R_{\max},z)=0 \qquad (13)$$

The solution of the boundary (-volume) problem (12)-(13) is supposed to have the form:

$$\Theta(r,z)=\Psi(r)\cdot\Xi(z) \qquad (14)$$

In this case partial derivatives in the equation (12) can be substituted by regular derivatives:

$$\frac{\partial\Theta}{\partial z}(r,z)=\Psi(r)\cdot\Xi'(z)\,,\ \frac{\partial\Theta}{\partial r}(r,z)=\Psi'(r)\cdot\Xi(z)\,,\ \frac{\partial^2\Theta}{\partial r^2}(r,z)=\Psi''(r)\cdot\Xi(z)\,.$$

We substitute the derivatives in the equation (12) and divide both parts by $\Psi(r)\cdot\Xi(z)$. We obtain:

$$\frac{{}_z(r)}{}\cdot\frac{\Xi'(z)}{\Xi(z)}=\frac{1}{r}S(r)\frac{\Psi'(r)}{\Psi(r)}+\frac{dS}{dr}(r)\cdot\frac{\Psi'(r)}{\Psi(r)}+S(r)\frac{\Psi''(r)}{\Psi(r)} \qquad (15)$$

what can be transcribed in the form

$$\frac{\Xi'(z)}{\Xi(z)}=\frac{}{\Psi(r)\cdot{}_z(r)}\left[\frac{1}{r}S(r)\,\Psi'(r)+\frac{dS}{dr}(r)\,\Psi'(r)+S(r)\,\Psi''(r)\right]=-\ {}^2 \qquad (16)$$

where ε - is a certain constant.

We have two regular differential equations:

1) a linear uniform differential equation of the first-order: $\Xi'(z)+\ {}^2\Xi(z)=0$, having the solution $\Xi(z)=C\cdot\exp\left(-\ {}^2z\right)$;

2) a linear uniform differential equation of the second-order:

$$S(r)\Psi''(r)+\left[\frac{1}{r}S(r)+\frac{dS}{dr}(r)\right]\Psi'(r)+\ {}^2\frac{{}_z(r)}{}\Psi(r)=0 \qquad (17)$$

The equation (17) is supplemented with the boundary conditions, which are obtained from the conditions (13) by substituting in them $\Theta(r,z)$ into $\Psi(r)\cdot\Xi(z)$:

$$\Psi'(0)=0\,;\ \psi\,\Psi(R_{\max})=0 \qquad (18)$$

The equations (17)-(18) correspond to the notation of Sturm-Liouville boundary (-volume) problem [7], which is known to have uncommon solutions not with all values of ε2, but only with some, which are called boundary (-volume) problem eigenvalues (17)-(18). These values form an infinite countable set. Uncommon solutions $\Psi(r)$ corresponding to them are called equation eigenfunctions (17)-(18). The eigenfunctions are defined accurately within the constant, therefore, one more condition: $\Psi(0)=1$, can be added to the conditions (18).

We'll denote $X(r)=\frac{1}{r}S(r)+\frac{dS}{dr}(r)$. Now let's enter a nonuniform grid along the axis *r* similarly to the solution of the direct boundary (-volume) problem, describing heat transfer in a fluid flow. The grid pitch ${}_i=r_{i+1}-r_i$, where $i=\overline{0,\,M-1}$, $r_0=0$, $r_M=R_{\max}$. Then we obtain: $X_i=\frac{S_i}{r_i}+\frac{S_i-S_{i-1}}{2\ {}_{i-1}}+\frac{S_{i+1}-S_i}{2\ {}_i}$.

The equation (17) is approximated on the formed grid in the following manner:

$$S_i\left[\frac{2}{{}_{i-1}+{}_{i}}\left(\frac{\Psi_{i+1}-\Psi_i}{{}_{i}}-\frac{\Psi_i-\Psi_{i-1}}{{}_{i-1}}\right)\right]+X_i\left(\frac{\Psi_i-\Psi_{i-1}}{2\,{}_{i-1}}+\frac{\Psi_{i+1}-\Psi_i}{2\,{}_{i}}\right)+\Psi_i\frac{{}^{2}\;{}_{z}\left(r_i\right)}{}=0 \qquad (19)$$

The boundary conditions are taken down in the form:

$$\Psi_0=1\,;\ \Psi_1=\Psi_0\,;\ \Psi_M=0 \qquad (20)$$

The solution of the difference equations (19)-(20) can be obtained by the method of testing [6]. For this purpose let’s define some initial estimate ε2 and substitute it to the equation (19). This equation links the value of function $\Psi(r)$ at three adjoining grid pitches. After that, using the known Ψ_0 and Ψ_1 we can calculate Ψ_2, then using Ψ_1 and Ψ_2 we calculate Ψ_3 and so on up to Ψ_M. Reproducing this calculation with a different ε2, we finally obtain the condition $\Psi_M=0$. Whereas in the sequel we’ll be interested only in the first eigenvalue of Sturm-Liouville boundary (-volume) problem, we have to select the least ε2, which allows to fulfill the indicated boundary condition. Then the value ${}_1^2$ will be the first eigenvalue, while its corresponding function $\Psi_1(r)$ will be the first eigenfunction of Sturm-Liouville boundary (-volume) problem (17)-(18).

The general solution of the boundary (-volume) problem (12)-(13) has the form:

$$\Theta(r,z)=\sum_{n=1}^{\infty}C_n\cdot\Psi_n(r)\cdot\exp\left(-\ {}_n^2 z\right), \qquad (21)$$

where *Cn* - is certain constants; $\Psi_n(r)$, ${}_n^2$ - are thereafter the eigenfunctions and the eigenvalues of Sturm-Liouville boundary (-volume) problem; $n=1,\ 2,\ 3,\ \ldots$ when analysing the solution of the boundary (-volume) problem (12)-(13) it is determined that in the formula (21) all the terms of the series, except the first, are negligible at $z\ge z^*_{\min}$, where $z^*_{\min}$ - is a certain threshold value, depending on a flow speed (Rainold’s number), an internal tube diameter, thermophysical fluid characteristics and an allowable value of error. Then with $z\ge z^*_{\min}$ the formula (21) can have the form:

$$\Theta(r,z)=C\cdot\Psi_1(r)\cdot\exp\left(-\ {}_1^2 z\right) \qquad (22)$$

Let’s find the logarithm of the both equation parts (22):

$$\ln\left[\Theta(r,z)\right]=\ln\left[C\cdot\Psi_1(r)\right]-\ {}_1^2 z \qquad (23)$$

then differentiate it with *z* and express the quantity ${}_1^2$ from it:

$${}_1^2=-\frac{d\ln\left[\Theta(r,z)\right]}{dz} \qquad (24)$$

According to the equation (22), the logarithm of the dimensionless temperature $\Theta(r,z)$ linearly depends on the coordinate *z*, therefore in the formula (22) we can pass from the derivative to the finite differences:

$${}_1^2=\frac{\ln\left[\Theta(r,z_1)\right]-\ln\left[\Theta(r,z_2)\right]}{z_2-z_1} \qquad (25)$$

but it’s necessary to fulfill: $z_1\ge z^*_{\min}$ and $z_2\ge z^*_{\min}$.

These formulae are the basis of the second method of turbulent Prandtl number ***Prtb*** defining. The essence of the method is obtaining from the experiment at least two temperature values $U(r,z_1)$ and $U(r,z_2)$, which are translated into the dimensionless temperature $\Theta(r,z_1)$ and $\Theta(r,z_2)$, and then are substituted into the formula (25). In this way the solution of Sturm-Liouville boundary (-volume) problem (17)-(18) is obtained. Defining a certain initial estimate ***Prtb***, we solve the system of difference equations (19)-(20), using previously described method of testing. Only now we modify not an eigenvalue ${}_1^2$, which we know from the experiment, but we modify the turbulent Prandtl number ***Prtb***, being in the formula for $S(r)$. Finally, we select such value of this number, which satisfies the boundary condition to the right $\Psi_M=0$ and function $\Psi(r)$ doesn’t take a negative value in the range $[0,\ R_{\max}]$, i.e. that the defined from the experiment eigenvalue of Sturm-Liouville boundary (-volume) problem is the first. Then we can conclude, that the fitted value ***Prtb*** is the true value of the turbulent Prandtl number for the given fluid flow.

When measuring the temperatures $U(r,z_1)$ and $U(r,z_2)$ it’s necessary to choose *z1* and *z2*, so as, first, they were not less than $z^*_{\min}$, secondly, the first and the second points are situated at a considerable distance from each other. Otherwise, the error, caused by the uncertain measurement of temperature, will affect much.

To decrease errors it's necessary to obtain several values of the temperature $U(r, z_j)$, (at different sections), where $j = \overline{1, N}$, to translate them into the dimensionless temperatures $\Theta(r, z_j)$, and then using them, calculate $_1^2$, applying the method of the least square (MLS), with the formula [3]:

$$ {}_1^2 = \frac{\sum_{j=1}^{N} z_j \cdot \sum_{j=1}^{N} \ln\left[\Theta(r, z_j)\right] - N \cdot \sum_{j=1}^{N} z_j \ln\left[\Theta(r, z_j)\right]}{N \cdot \sum_{j=1}^{N} z_j^2 - \left(\sum_{j=1}^{N} z_j\right)^2} \qquad (26)$$

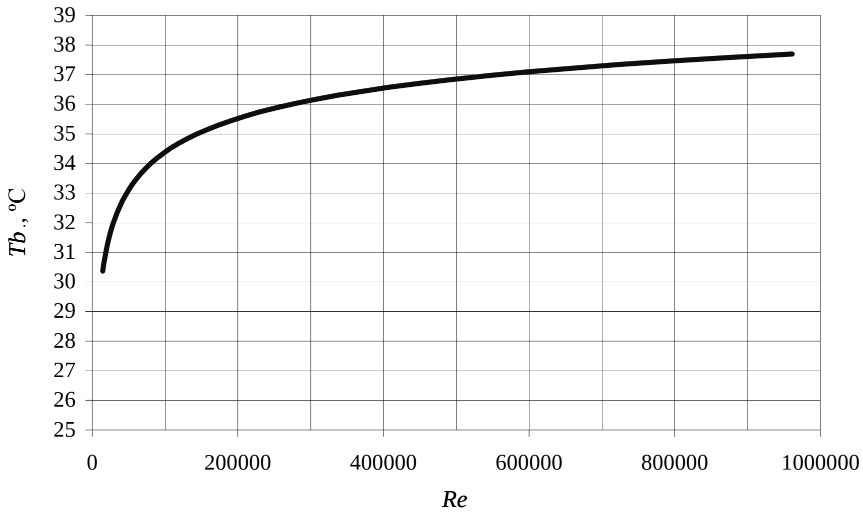
In the course of temperature measuring at different tube sections 2 (see Picture 1) with the help of a dipping device 5, it's necessary to remember that the average flow velocity $\overline{\ }$, and correspondingly, Rainold's number ***Re***, mustn't change. For this purpose, fluid pressure at a tube entry is necessary to regulate.

This method, as the previous one, is the basis of the computer program, which allows to calculate Prandtl number by the obtained from the experiment first eigenvalue of corresponding Sturm-Liouville boundary (-volume) problem.

5. The results of the numerical simulation

The designed computer programs can be used not only during the experiment, but during the stage of simulation as well, in order to specify the geometry of the plant (unit) and to choose optimal regime characteristics of the experiment.

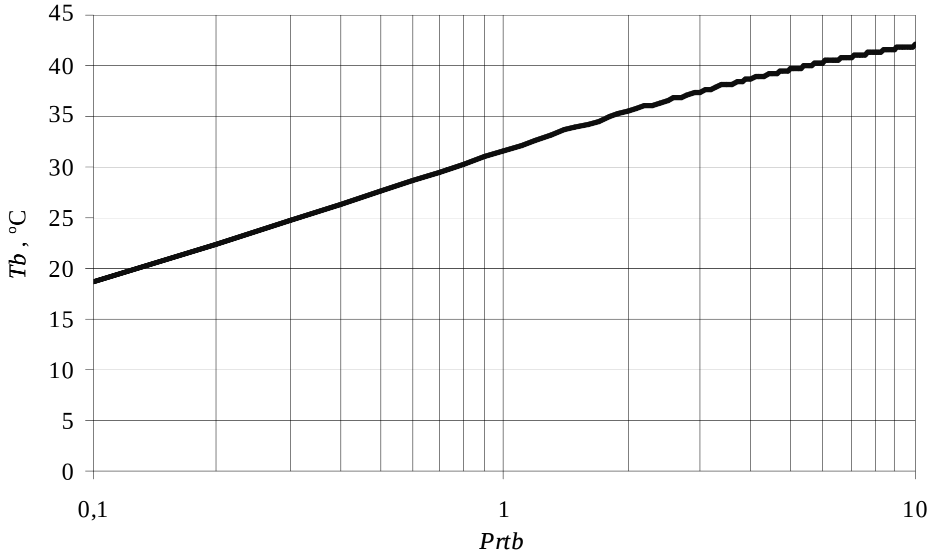
For example, Picture 2 shows the dependence of the bulk fluid temperature at a tube end upon Rainold's number, obtained from the solution of the direct boundary (-volume) problem (1)-(2). The length ***L*** of the division of the heat-exchange tube is taken as 1.5 m, the internal diameter is 10 mm. The water is supposed to flow in the tube and the turbulent Prandtl number is equal 1. The initial fluid temperature is equal 50˚C, the wall temperature is 10˚C. When calculating the velocity profile and the turbulent transfer momentum factors Reihardt formulae are applied.



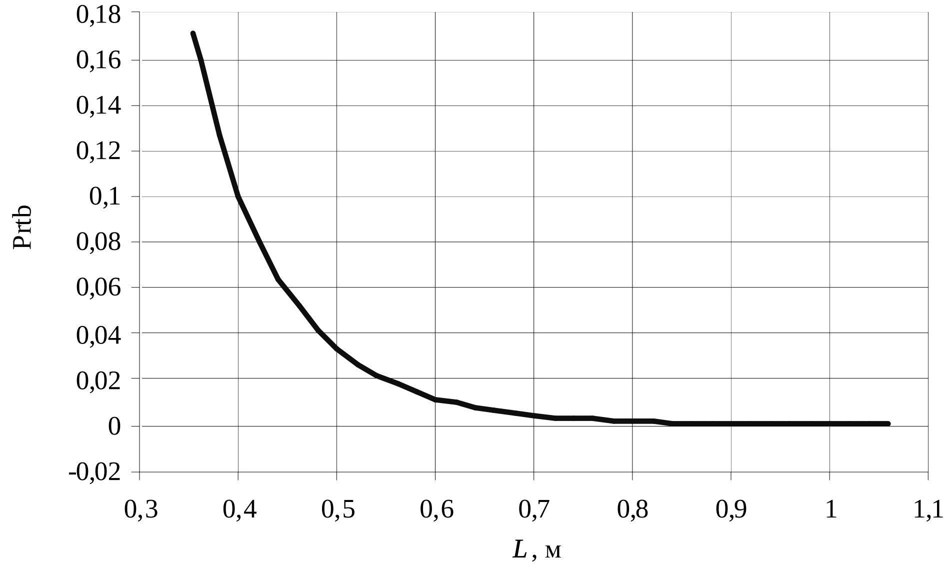
Picture 2. *The dependence of the bulk fluid temperature Tb at a measuring tube end upon Rainold's number Re*

The next Picture (3) shows the dependence of the bulk fluid temperature at a tube end upon Prandtl number, under the assumption, that all the rest flow parameters remain constant. All sizes and characteristics are the same as in the previous case. The fluid flow rate is 2·10-4 m3/c, which corresponds to ***Re*** = 25465.

Picture 4 shows the dependence of an absolute error of testing the turbulent Prandtl number upon the length of the heat-exchange tube division ***L*** under the assumption that the error of temperature measuring is equal zero. To build the graph we substituted different values of the tube length in the computer program, being used to solve the direct boundary (-volume) problem (1)-(2), i.e. to search out the temperature in the fluid flow. For that purpose we specify $\Pr_{тб} = 1$. From the identified temperature the first eigenvalue of Sturm-Liouville boundary (-volume) problem was calculated at a tube end ($z_1 \approx z_2 = L$) from the formula (25) and substituted in the computer program for the solution of the turbulent Prandtl number by the known $_1^2$; then an absolute deviation of the calculated ***Prtb*** from 1 was searched out.



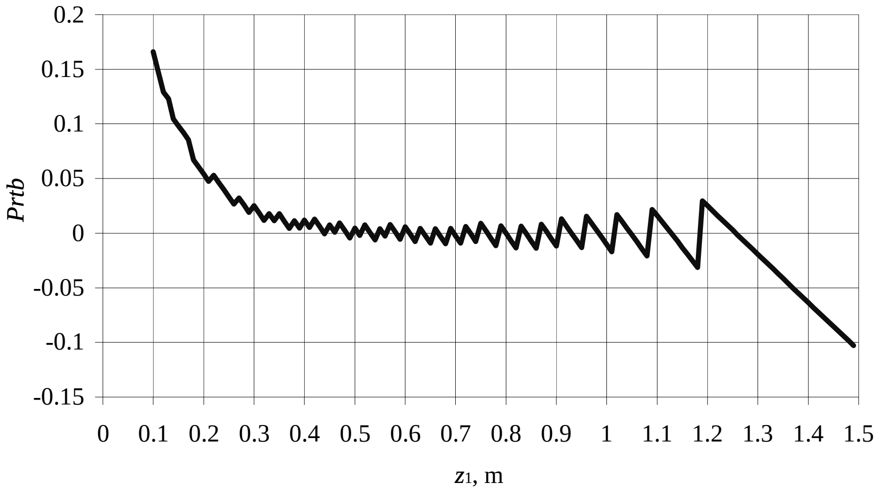
Picture 3. *The dependence of the bulk fluid temperature Tb at a measuring tube end upon the turbulent Prandtl number Prtb*



Picture 4. *The dependence of an absolute error ∆Prtb upon the length of the heat-exchange tube division L under the assumption of the precise temperature measurement*

On the basis of Picture 5 we can conclude that to obtain a minimum error, the first temperature $U(r,z_1)$ must be measured at a distance of 0,5-0,7 m from the entry of the heat-exchanger. When the fluid flow rate increases an optimal region for *z1* is shifted a little to the site of the increasing axis *z*. When *z2* is constant, the error is increasing.

The results obtained in the article are supposed to be used in choosing the design and regime values of the setting and measuring methods of the turbulent Prandtl number.



Picture 5. *The dependence of an absolute error ΔPrtb upon the axis z1 of the first point where the temperature is measured (when z2 = 1,5 m)*

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ИСКУССТВЕННЫЕ НЕЙРОННЫЕ СЕТИ. ПРОБЛЕМЫ И ПЕРСПЕКТИВЫ РАЗВИТИЯ©

Введение

В последние десятилетия в мире бурно развивается новая прикладная область математики, специализирующаяся на искусственных нейронных сетях (ИНС). Актуальность исследований в этом направлении подтверждается массой различных применений ИНС. Это автоматизация процессов распознавания образов, адаптивное управление, аппроксимация функционалов, прогнозирование, создание экспертных систем, организация ассоциативной памяти и многие другие приложения. С помощью ИНС можно, например, предсказывать показатели биржевого рынка, выполнять распознавание оптических или звуковых сигналов, создавать самообучающиеся системы, способные управлять автомашиной при парковке или синтезировать речь по тексту. В то время как на западе применение ИНС уже достаточно обширно, у нас это еще в некоторой степени экзотика - российские фирмы, использующие ИНС в практических целях, наперечет [1].

Термин «нейронный сети» сформировался к середине 50-х годов XX века. Основные результаты в этой области связаны с именами У. Маккалоха, Д. Хебба, Ф. Розенблатта, М. Минского, Дж. Хопфилда.

Некоторые исторические факты

В 1982-1985 гг. Дж. Хопфилд (J. Hopfield) предложил семейство оптимизирующих нейронных сетей, моделирующих ассоциативную память, появляются первые коммерческие нейрокомпьютеры.

В 1997 г. годовой объем продаж на рынке ИНС и НК превысил 2 млрд. долларов.
